Lecture 10/18/23 : Logs and Exponential models

Quiz 6 this Friday

HW 28 29 30
Sund M W

$\underbrace{\hspace{6em}}$ things we will cover this week !! :)

Last time we talked about how logs. They are

- the inverses to exponentials
- help bring powers down
- help solve exponential equations.

set up before class

Properties of Log

1) $\log_b(1) = 0$
2) $\log_b(b) = 1$
3) $\log_b(xy) = \log_b(x) + \log_b(y)$
4) $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
5) $\log_b(x^k) = k\log_b(x)$ — bring powers down
6) $\log_b(b^y) = y$
7) $b^{\log_b(x)} = x$

6), 7): inverse properties.

6) logs get rid of exp
7) exp. get rid of logs.

Ex: Solve for x : $3^x = 6$

Sol: $3^x = 6 \rightarrow \ln(3^x) = \ln(6) \rightarrow x\ln(3) = \ln(6)$

$\rightarrow \boxed{x = \dfrac{\ln(6)}{\ln(3)}}$

Doubling time:

Defn: The doubling time for an exp. equation is the amount of time it takes to double the initial amount.

There is a formula for this, but lets do an example first (2)

Not super enlightning ; we can use logic to get it in general

Ex: You have deposited $100 into a bank accunt w/ annual interest rate 5%. Find an equation for the amount of money you have in t years. Find the doubling time.

Sol: $P(t) = 100(1.05)^t$

To find doubling time we need to find time it takes to get $2 \times 100 = 200$ dollars. So solve

$$200 = 100(1.05)^t$$

$$2 = (1.05)^t$$

take powers down!

$$\ln(2) = \ln(1.05)^t$$

$$\ln(2) = t\ln(1.05)$$

$$\boxed{\frac{\ln(2)}{\ln(1.05)} = t}$$ ← Doubling time.

Formula for doubling time: To find doubling time of $A(t) = ab^t$ solve:

$$2 = b^t$$

only makes sense if $b > 1$ so we can grow to 2

Exactly what we did above

for t.

Half-time

Defn: The half time for an exp. equation is the amount of time it takes to get half the initial amount

There's a formula, but we will do an example first.

Ex: $P(t) = 1000(0.5)^t$ is the amount of bacteria in an exper. after t hours. What is the half life of the bacteria?

~~2840~~ $\frac{1}{2} \cdot 1000 = 500$

Solve: $500 = 1000(0.5)^t$

$\frac{1}{2} = (0.5)^t$

$\ln(\frac{1}{2}) = \ln((0.5)^t)$

$\ln(\frac{1}{2}) = t\ln(0.5)$

$\boxed{1 = \frac{\ln(\frac{1}{2})}{\ln(0.5)} = t}$

Formula for half life: To find the half live of an exponential $P(t) = ab^t$ $0<b<1$ solve

$\frac{1}{2} = b^t$ for t.

Ex: $f(t) = 40e^{-0.12t}$. Write as ab^t. What is its percent annual growth/decay rate?

Sol: $f(t) = 40e^{-0.12t} = \boxed{40(0.9801)^t}$

plug $r = 0.9801 - 1 = \cancel{.99}$
$= -0.0199$

decay rate of $\boxed{1.99\%}$

Ex: $g(t) = 330(1.23)^t$ write $g(t)$ in $y = ae^{kt}$

Sol: We want $330(1.23)^t = ae^{kt}$

Set $t=0$ initial so $a = 330$ [$330 = a$]

$$330(1.23)^t = 330e^{kt}$$

$$(1.23)^t = e^{kt}$$ Get rid of powers! Pick your favorite log

$$t\ln(1.23) = kt\ln(e)$$

$$\boxed{\ln(1.23) = k}$$

So

$$g(t) = ae^{kt} = 330e^{\ln(1.23)t}$$

Quicker way → $(= 330(1.23)^t)$

use # 7 on properties.

Challenging: 6, 7 ← similar to 6.

hint a): $p(t) = ab^t$

$2a = ab^5$

↑ doubling time 5 years

find b.

hint b): similar to part a

$p(t) = ae^{kt}$

solve for k

hint: We don't need to know a.